

A position dependent atom-atom entanglement in real-time Cavity QED system

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We study a special two-atom entanglement case in assumed Cavity QED experiment in which only one atom effectively exchanges a single photon with a cavity mode. We compute diatom entanglement under position-dependent atomic resonant dipole-dipole interaction (RDDI) for large interatomic separation limit. We show that the RDDI, even which is much smaller than the maximal atomic Rabi frequency, can induce distinct diatom entanglement. The peak entanglement (PE) reaches a maximum when RDDI strength can compare with the Rabi frequency of an atom.

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INTRODUCTION

Recently, generation of entanglement in Cavity QED system has been intensely paid attention to because of the motivations in the potential applications in quantum information [1, 2, 3] and computation processing [4, 5]. The realization of atomic entangled states in Cavity QED turns out to be feasible and fascinating since the experimental realization of single cold atoms in real-time Cavity QED [6]. A number of schemes of generating entanglement between cold atoms have been put forward to realize quantum teleportation [7, 8, 9] and swapping [10, 11]. In current investigated cold-atom schemes, atoms are trapped in optical cavities or magneto-optical traps (MOT) so that they can be connected through exchanging single photons with cavity (in the large interatomic separation regime) [12, 13, 14] or through resonant dipole-dipole interaction (RDDI) (in the small interatomic separation regime)[15, 16], and then be strongly entangled at a special interacting time. Furthermore, in case of weak atom-field coupling and large detuning between atomic transition and cavity frequency, even in the large interatomic separation regime, the Rabi oscillation of atom-field can be effectively treated as atomic RDDI which can certainly induce diatom entanglement [17]. So, in an assumed optical cavity, the atomic entangled states are generated from the collective contribution of Rabi coupling (RC) and RDDI. The competition between them leads to our optimal choice of the effective model. It has been demonstrated that, in case of diatom scheme, the asymmetric atom-field position-dependent RC ($g_1 \neq g_2$ with $g_{1(2)}$ the Rabi frequencies) depresses the fidelity of the maximal entanglement (ME)[12]. In fact, if maximally asymmetric atom-field RC (MARC) emerges, as $\frac{g_1 g_2}{g_1 + g_2} = 0$, Rabi frequency can never induce any atomic entanglement. While, under this circumstance, if we involve the RDDI between atoms, the entanglement situation can be different even when the RDDI is weak enough. In this paper, we investigate such an assumed experimental situation. We will show how to generate MARC in a Cavity QED system and what is the diatom entanglement

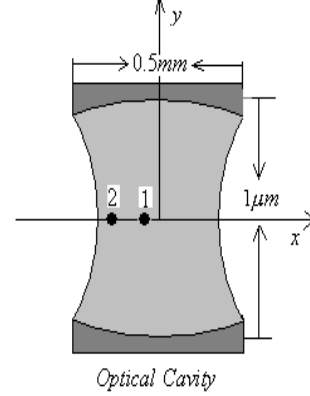


FIG. 1: Schematic diagram for proposed model. One atom is trapped in an optical cavity, the other is located not far away beside the former.

under various competition between RC and RDDI in it.

MODEL DESCRIPTION

Here we consider a system constituted by two-level cold atoms 1 and 2 that couple to a single mode electromagnetic cavity field which is assumed to be a Gaussian mode profile, as is shown in Fig. 1. The whole experimental apparatus is described as follow: Two super-polished spherical mirrors of radius of curvature 1cm constructed a cavity of length 1um with cavity waist $w_0 \sim 4\text{um}$. In this cavity, the maximum atom-field coupling coefficient that only occurs at an antinode of the cavity field mode is $g_0 \simeq 400\text{MHz}$. We choose two atomic levels are $6S_{1/2}$ and $6P_{3/2}$ of Cesium atoms with corresponding transition wavelength $\lambda \simeq 850\text{nm}$. We denote the interatomic separation as R and the wave vector of the atomic emitted photon as k_0 .

In the rotating-wave approximation (RWA), and in the interaction picture in case of resonant coupling, the Hamiltonian governing this system reads

$$H = \sum_i g_i (a \sigma_i^+ + h.c.) + \Gamma (\sigma_1^+ \sigma_2^- + h.c.) \quad (1)$$

where $\sigma_1^z, \sigma_2^z, \sigma_1^\pm$ and σ_2^\pm are spin operators and raising (lowering) operators of atom 1 and 2 respectively, a^+ (a) is the creation (annihilation) operator of field, g_1 (g_2) is the coupling strength of atom 1 (2) to field respectively, while Γ is

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the coefficient of atom-atom RDDI which is included when the interatomic separation R is not much too large [15, 16]. It has been noted that both g_i and Γ are position-dependent variables for Cavity QED theoretically and experimentally. So that $g(r) = g_0 \cos(2\pi x/\lambda) \exp[-(y^2 + z^2)/w_0^2]$ [18], w_0 is the length of the cavity waist, and Γ has a complicated form that depends on $\frac{1}{R}$, $\frac{1}{R^2}$, and $\frac{1}{R^3}$. The completely solving of such a system is possible but complicated. Here we consider a physically possible and feasible circumstance, where two atoms enter the cavity in turn, the separation between them is in the order of cavity waist. We also assume the atomic velocity is much smaller so that the typical atomic transit time scale is much larger than the interacting time scale. So the Hamiltonian in Equ. (1) is time-independent. The solving of this circumstance is much more simply and neatly. We assume atom 2 is located in a position about $x_2 \simeq -5w_0$, atom 1 is trapped in the region not far from the antinode of the field, so that $-2w_0 \leq x_1 \leq 2w_0$. As a result, $g_2 \sim 10^{-10}g_1$. This approximation leads to a MARC Hamiltonian

$$H = g_1(a\sigma_1^+ + h.c.) + \Gamma(\sigma_1^+ \sigma_2^- + h.c.) \quad (2)$$

In the invariant sub-space of the global system, for the system of only a single photon shared by atom pairs and cavity field, the Hamiltonian can be written on a set of complete basis $|g, g, 1\rangle, |e, g, 0\rangle, |g, e, 0\rangle$ as

$$H = \begin{pmatrix} 0 & g_1 & 0 \\ g_1 & 0 & \Gamma \\ 0 & \Gamma & 0 \end{pmatrix} \quad (3)$$

The eigenvalues of this Hamiltonian can be obtained as $E_1 = 0$, $E_{2,3} = \pm g_1 \sqrt{1 + \gamma^2}$, where $\gamma = \frac{g_1}{\Gamma}$. The corresponding eigenvectors are obtained as

$$\begin{aligned} |\phi_1\rangle &= \xi_1[\gamma|g, g, 1\rangle + |g, e, 0\rangle] \\ |\phi_{2,3}\rangle &= \xi_{2,3}\left[\frac{1}{\gamma}|g, g, 1\rangle + |g, e, 0\rangle \right. \\ &\quad \left. \pm \sqrt{1 + \frac{1}{\gamma^2}}|e, g, 0\rangle\right] \end{aligned} \quad (4)$$

with ξ_i the normalized factor. The first eigenstate $|\phi_1\rangle$ with corresponding zero eigenvalue, describing atom one fixed on its ground state, can be seen as a "dark state".

For a given initial state $|\psi(0)\rangle$ of system, we can obtain the evolved dressed state $|\psi(t)\rangle$ of system which can be expanded as a superposition of eigenstates $|\phi_i\rangle$

$$|\psi(t)\rangle = \sum_i C_i(t) |\phi_i\rangle \quad (5)$$

The coefficients $C_i(t)$ are obtained by solving Schrödinger Equation, so that $C_i(t) = C_i(0)e^{-iE_it/\hbar}$ with $C_i(0)$ determined by initial conditions. After a sub-Doppler and evaporating cooling suffering, atom 2 are reasonably on their ground state, i.e. $6S_{1/2}$. While, atom 1 might in a superposition of its ground state and excited state due to the strong correlation

between it and cavity field. The resulting initial system state is $|\psi(0)\rangle = |g\rangle_2 \otimes (\alpha|g\rangle_1|1\rangle + \beta|e\rangle_1|0\rangle)$. The system state is then given by

$$\begin{aligned} |\psi(t)\rangle &= (-\mu\alpha - \nu\beta)|\phi_1\rangle + [(\mu\alpha + \nu\beta)/2 \\ &\quad - \frac{\nu}{\mu}\gamma\beta]e^{-iE_2t}|\phi_2\rangle + (\mu\alpha + \nu\beta)/2e^{-iE_3t}|\phi_3\rangle \end{aligned} \quad (6)$$

where $\mu = \frac{\gamma}{(1+\gamma)^2}$, $\nu = \frac{\gamma}{(1+\gamma)^2}$.

The reduced density matrix of diatom is obtained by tracing over the field variables of system density matrix $\rho(t) = |\psi(t)\rangle\langle\psi(t)|$, that is $\rho_{atom}(t) = \text{Tr}_f \rho(t) = \sum_n \langle n|\rho(t)|n\rangle$. From Equ. (6), the atomic density matrix is

$$\rho_{atom}(t) = \begin{pmatrix} D & 0 & 0 & 0 \\ 0 & B & E & 0 \\ 0 & E & C & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (7)$$

where $E^2 = B * C$. The off-diagonal element E describing the correlation between atomic transition $|e\rangle_{1(2)} \rightarrow |g\rangle_{1(2)}$ and $|g\rangle_{2(1)} \rightarrow |e\rangle_{2(1)}$ represents the strength of the entangled state for two atoms. In the next section, we will discuss the entanglement induced by the RDDI between atoms.

DIATOM ENTANGLEMENT NATURE UNDER CAVITY FIELD

Wootters Concurrence, which has been proved to be effective in measuring the entanglement of two qubits, is defined as[19]

$$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \quad (8)$$

where λ_i are four non-negative square roots of the eigenvalues of the non-Hermitian matrix $\rho(\sigma_y \otimes \sigma_y) \rho^*(\sigma_y \otimes \sigma_y)$ in decreasing order. In dealing with this model, the Concurrence is simply determined by several density matrix elements since most of the off-diagonal elements are eliminated due to the adiabatic evolution. In our case, the Concurrence can be proved to be exactly twice the absolute none-zero off-diagonal element as $C(\rho(t)) = 2|E(t)|$.

For large interatomic separation R , which corresponding the far-zone case with $Rk_0 \sim 10^2 \gg 1$, the dipole-dipole interaction strength can be estimated as $\Gamma \sim \omega_a a_0^3 k_0^2 / R$ [15, 20], where c is light velocity, a_0 is the atomic excited state radius. This emerges the limit of the RDDI strength as $\Gamma \sim 10^5 H_z \leq 10^{-2}g_1$. Our further assumption is the atom that earlier enters the cavity is now also in its ground state, i.e. $\beta = 0$ in Equ. 6. (Certainly, this assumption leads to the half possibility of the successively entangling two atoms.)

The diatom entanglement shows two-peak split structure. Each peak corresponds to a PE which occurs at $t = \frac{(3m+1)\pi}{3} \frac{1}{\sqrt{g^2 + \Gamma^2}}$ with $m = 1, 3, 5 \dots$ (see Fig. 2). The increase of time leads to an entanglement period that depends on the Rabi

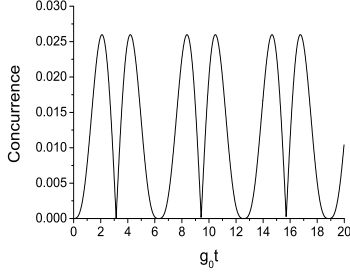


FIG. 2: The evolution of diatom entanglement versus time, where the location of atom 1 is $x_1 = -2w_0$

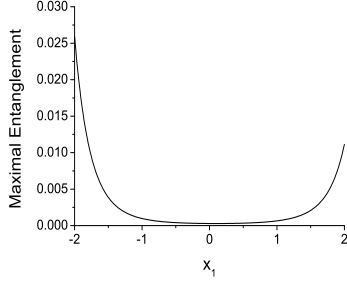


FIG. 3: The diatom PE occurs at $t = \frac{2\pi}{3} \frac{1}{\sqrt{g^2 + \Gamma^2}}$ versus the location of atom 1 x_1 (in unit of cavity waist w_0).

frequency of atoms and RDDI strength. Both the amount and period of PE are position-dependent. When the initial location of atom 1 departs from atom 2 gradually, they decay exponentially and fall to their minimum at the center of the cavity, see Fig. 3 and Fig. 4. These can be understood since Rabi frequency increases exponentially in this process and reaches its maximum at the center of the cavity, while RDDI strength decreases polynomially. Similarly, as the initial location of atom 1 increases along x axis gradually from the center of cavity, the amount and period of PE can be recovered since Rabi frequency decreases now.

In fact, from Equ. (6) we can see that, the amplitude of the entanglement is only determined by the ratio of RDDI and Rabi frequency of atom 1. In the region of $w_0 \leq |x_1| \leq 2w_0$, the ratio (namely γ) is in the order of 10^{-3} . In this region, the effect of RDDI can not be neglected since the PE is distinct, even the interatomic separation can approach $7w_0$.

While, in the region of $|x_1| \leq w_0$ (which corresponds to a fast-oscillating regime), the coupling between atom 1 and cavity field is so much strong that the ratio $\gamma \approx 0$. The Hamiltonian in Equ. (2) can be adiabatically written as

$$H_{eff} = g_1(a\sigma_1^+ + h.c.) + \frac{2\sqrt{2}\Gamma^2}{g_1}(\sigma_1^z\sigma_2^+\sigma_2^- - \sigma_2^z\sigma_1^+\sigma_1^-) \quad (9)$$

The system described by this Hamiltonian can never generate entanglement since the diatom coupling part in Equ. (9) is diagonalized.

In order to give an overall description of the diatom entan-

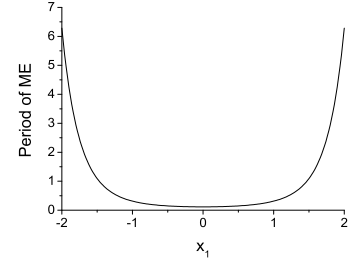


FIG. 4: The period of PE (in unit of inverse of g_0) versus the location of atom 1 (in unit of cavity waist w_0).

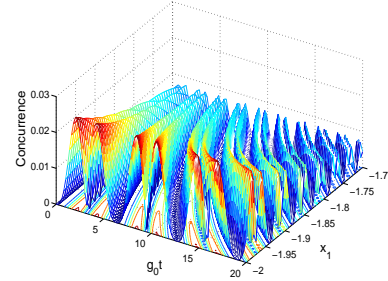


FIG. 5: The evolution of diatom entanglement versus time and the location of atom 1 (in unit of cavity waist w_0).

glement, we mesh the concurrence under the position of atom 1 and time in Fig. 5. From this figure, we can clearly get the hint of two-peak split structure and the period of PE.

All the results reveal that, in the weak Rabi coupling region, the amount and the period of PE is larger than that in other region. Since the participation of atom-field interaction results in diatom mixed state which can not be an exactly Bell-state, the quality and quantity of PE is strongly depressed.

CONCLUSION

We investigate a diatom entanglement case for maximally asymmetric Rabi coupling in Cavity QED. The characterized apparatus parameters are constituted for experimental Optical Cavity. The entanglement situation can be analyzed bases on the ratio of position-dependent RDDI strength and Rabi frequency of atom 1. We concretely point out the amplitude (which is a two-peak split structure) and the period of the entanglement in two regions. In the region of relative large ratio, where the RDDI strength can be in the order much larger than $10^{-3}g$, diatom entanglement can be distinctly generated. We do not involve the motion of center-of-mass of two atoms. While, if the atomic velocity is large enough and the typical transit time scale can compare with the interacting time scale of the system, the effect of the motion on the entanglement must be included. And the Hamiltonian will be time-dependent one. Additionally, our MARC approximation

should be still effective for wider initial location of atom 1. Even if atom 1 is initially located at $x_1(0) = \pm 3w_0$, where the RDDI strength would be in the same order of Rabi frequency and the ratio γ can approach 1.2, the validity of this approximation keeps well since $g_1 \sim \Gamma \sim 10^7 g_2$. It should be pointed out that, actually, an underlying trouble for the fidelity of the results here may be the cavity dissipation and the atomic spontaneous emission that can lead to an exponential decay to the excited state population. But, in consideration of the high-Q regime of the cavity and the fact of cavity compressing atomic spontaneous emission for not two large interatomic separation [21], in a short-time range of the evolution, the trouble may be faint and nonsignificant.

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